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ABSTRACT

Examined is the notion of "representation" as it is used in mathematics education. The first section explains what appears to the current "received view" of representation within mathematics education. The second section describes the constructivist view of representation, especially as this view is related to Piaget's theory. The third section examines features of the information processing view of representation. The fourth section analyzes some of the fundamental differences between constructivist and information processing views. These differences focus on: (1) the use of representation as "picture of knowledge"; (2) the role of mental activity in each representation; (3) the difference between a functional and a functional-structural view of representation; and (4) the relationship between logic and psychology in each model. The ultimate goal of the paper is to contrast constructivist and information processing views of representation in mathematics education. (PK)

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CONSTRUCTIVIST AND INFORMATION PROCESSING VIEWS OF REPRESENTATION IN MATHEMATICS EDUCATION

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CONSTRUCTIVIST AND INFORMATION PROCESSING VIEWS OF REPRESENTATION IN MATHEMATICS EDUCATION

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The notion of "representation" is ambiguous (Kaput, 1985; von Glasersfeld, 1987). In some work in mathematics education, "representation" refers to a formal model of a learner's knowledge (cf., Anderson et. al., 1981). In its artificial intelligence sense, "repesentation" is a formal system that can be implemented on a machine. Other mathematics educators use "representation" to refer to the product of the child's cognitive activity. Some constructivists, for example, spell the term "re-presentation", to emphasize the fact that the child is actively representing or constructing objects during her learning (von Glasersfeld, 1987; Cobb & Steffe, 1983). The present paper is a further examination of the notion of "representation" as it is used in mathematics education.

The structure of this paper is as follows. The first section will explain what appears to be the current "received opinion" of representation within mathematics education. This will be used as a backdrop to further refine and analyze the use of the concept of "representation". The second section will describe the constructivist view of representation, especially as this view is related to Piaget's theory. The third section will examine features of the information processing view of representation. Finally, the fourth section will analyze some of

the fundamental differences between constructivist and information processing views. These differences will focus on:

- the use of representation as "picture of knowledge" and as "object of knowledge"
- the role of mental activity in each view of representation
- the difference between a functional and a functional-structural view of representation
- the relationship between logic and psychology in each model

The ultimate goal of this paper is to contrast constructivist and information processing views of representation in mathematics education.

The "Received View" of Representation in Mathematics Education

The way in which the term "representation" is most ordinarily used in mathematics education is as a particular embodiment or interpretation of a mathematical concept or idea. For example, Lesh et. al. use "representations" as "external (and therefore observable) embodiments of students' internal conceptualizations..." (Lesh, Post, & Behr, 1987, p. 33). Mathematics educators speak of being able to "represent" the concept of place value using multibase blocks, place value charts, or colored chips. A "representation" of an idea, in this sense, is an embodiment or interpretation of the idea.

Kaput (1985, 1987), following others, notes that several things must be specified when talking about a representation:

(a) a represented world;

(b) a representing world;

(c) aspects of the former which are represented;

(d) aspects of the latter which do the representation;

(e) the correspondence between them.

A representation might be compared to a mathematical function. As with specifying a function, specifying a representation requires a domain (the thing being represented), a range (the thing doing the representation), and a rule linking the two. In representing the place value concept with multibase blocks, for example, the domain would be the mathematical concept, the range would be the blocks, and the rule would be that the concrete embodiment must somehow help the child come to understand the place value concept.

"Representation" is also used in mathematics. Kaput argues that "the idea of representation is continuous with mathematics itself" (Kaput, 1987, p. 25). One of his several examples is Cayley's Theorem, "which states that any group is isomorphic to a group of permutations on its elements." (ibid., p. 24). Here the representing entity (domain) is the permutation group, the represented entity (range) is the original group, and the relationship (rule) is the isomorphism. "Representation", in this sense of the word, is a formal connection between one mathematical entity and another.

The sense that representation is used in mathematics education might be a first cousin of the sense in which the word

is used in mathematics. However, there are some family differences. First, the domain of the representation in mathematics education (eg., "the learner's concept of place value") does not belong to the subject matter of mathematics. This element is a psychological rather than a mathematical construct. Second, the relationship between domain and range is less formal in the case of mathematics education. When a mathematics educator speaks of the concept of place value as being represented by multibase blocks, she means, to some extent, that here is a formal or logical relationship between the two notions. However, this relationship might not be specified by an isomorphism in the mathematical sense. There is an added factor in the relationship between place value and the multibase blocks that does not exist in the case of the relationship between a group and the group of permutations of its elements. The added factor pertains to mathematical understanding. The multibase blocks represent place value in the sense in that they help the learner better understand the concept of place value. In other words, the relationship between domain and range in mathematics education is that the range should have a formative impact on the domain.

In mathematics education, representation is not limited to the sense of a concrete embodiment. Dienes uses the term "representation" when describing the cognitive process of coming to understand an idea. Here the representation is not an external embodiment but a cognitive entity. A child might be

exposed to a number of physical embodiments of the place value concept and then begin to abstract this concept. In this process, the child builds an "internal representation". Dienes describes the situation as follows:

> Let us now assume that a certain number of such structures have been extracted from similar (isomorphic situations). How can we help the child to anchor this incipient abstraction? We need a peg on which to hang what he has abstracted. What is this peg? It is a representation. He needs a common representation by means of which he can see for himself what the communalities are which he has realized are to be found in all the embodiments (Dienes, 1960, p. ,italics in original).

"Representation" in this sense is not a physical embodiment of an idea. A representation is itself an idea.

The distinction between "embodiment" and "mental representation" might be used to name these two senses of representation in mathematics education. In an embodiment, the domain is a mental idea and the range is a physical analogue of the idea. The rule is that the embodiment should somehow help the child to better understand the mathematical idea. In a mental representation, the domain is a mathematical idea and the range is also an idea. Some constructivists argue that the "rule" linking the two is that the representing idea is a "reflective abstraction" of the representing idea.

To explore this issue in more detail, the constructivist idea of representation will be examined next.

The Constructivist View of Representation

Constructivists are interested in the processes that learners of mathematics used to create their representations. Here, "representation" is not used in the sense of a relationship between an external embodiment and a mathematical idea. In the constructivist sense, representation is a relationship among ideas. The domain of the representation consists of previous actions. The range might be described as an object before the child's "minds eye". One way to describe the rule is that the object before the child's mind's eye must be mentally constructed from previous actions.

Constructivists in mathematics education are heavily indebted to the work of Piaget. Perhaps the most fundamental idea that constructivists borrow from Piaget is that knowledge is intimately connected with action. Knowledge "arises from interactions that take place mid-way between [the child and her environment] and thus involves both at the same time..." (Piaget, 1972, p. 19). "... [E]very perception gives to the perceived elements meanings which are relative to action..., and it is therefore from action that we need to start..." (Piaget, 1972, p. 20). "The 'intuition' of space is not a 'reading' or apprehension of the properties of objects, but from the very beginning, an action performed on them". (Piaget & Inhelder,

1963, p. 449). "Logic in the child (as in the adult) is evidenced in the form of operational structures; i.e., the logical act consists essentially of operating, hence of acting on things and towards people (Piaget, 1968, p. 121). "... [E]ven perception calls for a subject who is more than just the theater on whose stage various plays independent of him ... are performed; the subject performs in, sometimes even composes these plays..." (Piaget, 1970, p. 59). "Intelligence organizes the world by organizing itself" (Piaget, 1954).

Different types of action give rise to different types of knowledge. Piaget contrasts knowledge obtained by perception and by representation.

> Perception is the knowledge of objects resulting from direct contact with them. As against this, representation or imagination involves the evocation of objects in their absence or, when it runs parallel to perception, in their presence (Piaget & Inhelder, 1963).

Thus, for Piaget, representation requires the further activity of evoking or constructing an object. Some constructivists in mathematics education (Cobb, Steffe, von Glasersfeld) spell "representation" as "re-presentation", to emphasize the evocation or constructive element.

For Piaget, representation has its origin in "imitation". In the case of spatial concepts, for example, representation begins with an image, which arises from the act of imitation:

> ... considered from the point of view of its origin, the image is a product of imitation. It is in fact, an internalized imitation, one that can be made without resort to external gestures... [and] is basically a continuation of the adaptive movement characteristic of actions. (ibid., p. 40).

Imitation provides the child with a representation of a previous event or process. By reflecting on this representation, the child constructs new knowledge. Piaget calls this process "reflective abstraction".

In all forms of representation, there is a difference between the thing being represented and the thing doing the representation. "What is distinctive of representation is a system of meanings or significations embodying a distinction between that which signifies and that which is signified" (Piaget & Inhelder, 1964, p. 17). In the case of spatial images, the thing doing the representing is the image and the things being represented are "spatial transformations, spatial states, etc." (ibid., p. 17). Expressed in the language used earlier, the signified is the domain and the signifier is the range. One way to describe the rule linking domain and range is that the signifier must somehow serve the child's intellectual functioning. In other words, the signifier must help the child "organize the world by organizing herself".

Constructivist in mathematics education pay special attention to the role of reflective abstraction in the learning

of mathematics. Proponents of another view of representation in mathematics education, however, focus less on the child's reflections on her actions and more on representing the child's behavior in a formal model. This information processing view will be considered next.

Information Processing Views of Representation

Representation, within a constructivist framework, is a representation of action. Actions, both sensori-motor and cognitive, form the raw materials for reflective abstraction and the construction of mental structures. Information processing (IP) theorists are also interested in a child's cognitive actions. However, often the underlying representation is a collection of propositional statements rather than a child's actions. The child is assumed to be "active"; however, activities are partly constrained to those that can be modeled by the action of a machine on a system of propositions.

The distinction between declarative and procedural knowledge, or knowledge that can be stated in propositional form and knowledge which is manifested in doing or acting, is important for many IP theories of representation. A student might be able to state the Pythagorean theorem, in which case she has declarative knowledge of this theorem. Or, the student might be able to solve problems with right triangles, in which case she has procedural knowledge of the theorem. IP theorists such as Anderson make heavy use of this declarative/procedural

distinction. Anderson represents a learner's knowledge as a network of linguistic propositions and a collection of formal rules that can generate new propositions from the existing network (Anderson, 1976). This formal model can (in principle) be implemented on a machine which will simulate a child's behavior.

One of Anderson's examples of a production system can help to indicate the formality of procedural knowledge. A production system for generating certain palindromes, for example, is as follows:

axioms: a,b,aa,bb

rules: (P1) $ -> a $ a

(P2) $ -> b $ b

where $ are arbitrary strings. A production of abaaaba can then be generated from the initial string, a, by following the productions (P1), (P2), (P1). (Anderson, 1976, p. 90). Minsky has shown that production systems are equivalent to Turning Machines in their power (ibid., p. 90).

Not all IP theorists agree that declarative and procedural representations of knowledge can be clearly distinguished. Rumelhart and Norman, for example, attempt to represent a learner's knowledge in "schemes" that are both propositional and procedural (Rumelhart and Norman, 1981). This is closer, in many ways, to the constructivist view of representation (cf. Cobb, 1977). For the purposes of contrast, however, the present discussion will not focus on IP theories such as those proposed

knowledge, that is, as a series of propositional statements in a network (ibid., p. 227). These propositional representations can be transformed into procedural representations by two different, hypothetical processes. In one process, called "knowledge compilation", specific actions and general operators are adduced from the propositional representations. In another process, called "subsumption", general operators embedded in the propositional representation are adduced which can then be used to solve specific problems (ibid., pp. 227-8).

There are similarities between production systems and S-R behaviorism (Anderson, 1976; Kaput, 1985). Both are "mechanical" in the sense that proponents of each attempt to represent complex knowledge in terms of specified primitives. The difference between the two theories is that, for S-R behaviorism, the primitives must be observable, whereas for production systems, the primitives can be internal processes. In both cases, however, one attempts to take a complex "unknown" (such as geometry problem solving) and mimic its input and output features using some "known" with postulated structural properties. The domain or unknown might be geometry problem solving. The range or known would be a declarative network plus production system. The rule or relationship between the two is that the unknown and known should "behave" similarly.

The IPI and the constructivist views of representation differ on several points. The following section will examine some of these differences.

by Rumelhart and Norman. The focus below will be on IP theories such as those put forward by Anderson wherein knowledge is represented as a propositional network and a production system. This latter theory of representation will be called the IP1 view, to distinguish it from other IP theories whose proponents draw a less radical distinction between declarative and procedural knowledge. Many of the contrasts between the IP1 and constructivist views of representation will not apply to other IP theories.

In the IP1 view of knowledge representation, the domain is a human being's knowledge of a particular area, and the range is an encoding of this knowledge in the form of propositional networks and procedures. The rule is a "functional" relationship between the human being's knowledge of the particular domain and its declarative/procedural representation. This "functional relationship" is that the human being should function or "behave similarly" to the outcome of the rules acting on the propositional network. In other words: When given the same inputs, the human being and the declarative/procedural network should give the same outputs.

One area in mathematics education where this IP1 view has been applied is in geometry problem solving (cf., Anderson et. al., 1981). The knowledge that students need to prove a theorem in geometry is represented as a network of "facts", such as those found in a geometry text, and as a network of solved geometry problems. Both of these networks are represented as declarative

Contrasts between IP1 and Constructivist Views of Representation

This section will examine four differences between the IP1 view of representation and the constructivist view. These pertain to:

- the use of representation as "picture of knowledge" and as "object of knowledge"
- the role of mental activity in each view of representation
- the difference between a functional and a functional-structural view of representation
- the relationship between logic and psychology in each model

- pictures of knowledge and objects of knowledge

IP1 theorists and constructivists views of representation can be distinguished by noting two senses of the word "representation". In one sense, "representation" is used as a description or model of something. It is in this sense that the solar system was, at one time, used to represent the structure of an atom. It is also in this sense that philosophers of science picture knowledge as a net or network (Hempel, 1966, p. 94) Concepts are represented by knots or nodes in the network, and relationships among these concepts are represented by filaments or strands. In this sense of "representation", the theorist is trying to give a picture or formal description of how a human being's knowledge might look. This might be described as a

picture of knowledge. A representation in this sense is similar to an "explanation" (Kaput, 1985).

A second sense of "representation" is as the product of some sort of mental activity. Kant uses this sense when he speaks of objects of our senses producing representations. As he puts it in the introduction to his Critique:

> "... objects affecting our senses partly of themselves produce representations, partly arouse the activity of our understanding to compare these representations..." (Kant, 1965, p. 41).

The concept of representation used by Kant is that the mind somehow presents to itself, or constructs, a certain object. Representation in this sense means something like "image". In other words, representation refers to the objects of knowledge.

The word Kant uses for "representation" is "Vorstellung", which is usually translated "idea" or "imagination" (cf, von Glasersfeld, 1987). German speakers and writers distinguish "Vorstellung" from another word for representation, "Darstellung". This latter word is related to the verb "darstellen", meaning "to show, depict, or delineate". It is often used in connection with theatrical performances.

Constructivists tend to be interested in representation as "Vorstellung". One of their goals is to discover how t. mind constructs or pictures mathematical ideas to itself. IP1 theorists may be interested in mental representation their prime interest would seem to be in a description or picture of

someone's knowledge. A semantic net of someone's mathematical knowledge would be a "Darstellung".

- the role of mental activity

The difference between representation as "Darstellung" (pictures of knowledge) and "Vorstellung" (objects of knowledge) is related to another difference between IP1 and constructivist views of representation. This is the sense in which the theorists study mental activity.

The role attached to mental activity is best explained by postulating a "rough continuum" between active and passive theories of the mind. At one end of the continuum is what might be called a passive theory of the mind. Greater importance is attached to the activity of the environment, rather than the mind. The human learner acquires knowledge of the world through the senses or through apprehension of eternal ideas. The focus, at this end of the continuum, is on what is "out there". Plato argued that the mind acquired knowledge by beholding eternal ideas or forms. Locke argued that Nature writes her story on the mind's "blank slate". Dewey (192-) referred to these as "spectator theories of knowledge".

At the other end of the continuum is what might be called an active theory of the mind. What is "out there" is conditioned by what we know. Supporters of this position argue that knowledge is constructed by the individual learner. Kant, who may have been the first proponent of this view, argued that:

> ... reason has insight only into that which it produces after a plan of its own... it must not allow itself to kept, as it were, in nature's leading-strings, but must itself show the way (Kant, 1965, p. 14).

Similarly, Piaget argues that the mind constructs and sometimes even "composes the play" which passes before the mind (Piaget, 1970, p. 59). At the extreme end of this continuum might be Hegel's view that World History is the unfolding of the Universal Spirit.

Though both IP1 and constructivist views of representation tend to lie towards to the "active end" of the above continuum, the constructivist view is more active. This can be seen by examining the relationship between the learner and the representation in the IP1 and the constructivist views.

IP1 theorists study learners' responses to certain cognitive tasks in order to create representations of inner processes. The data available to the IP1 theorist are often mathematical tasks and responses to these tasks. These data are used to construct a theory of the internal processes. One assumption of the theory is certainly that the child's mind is active; otherwise, no there would be no need to represent this activity using a declarative network plus a set of production rules. However, the representation ("Darstellung") is the active construction of the IP1 theorists' mind, not the child's mind. The learner's construction of representations is not as interesting as the representation of whatever constructive process is taking place

inside her mind.

This point could be expressed in a different way: IP1 theorists attempt to model or recreate the child's activity in a medium that is different from the child's mind (the IP1 model, or a machine). If, when realized on a machine, the model "behaves" in the same way that the child behaves, then the model suits its purpose. Thus, the model is primarily descriptive. "Why" a child behaves in a particular way is left more to an elaborate description of "how". In other words, the primary goal of the IP1 theorist would seem to be to represent the child's activity in terms of a model, not to explain the activity that is involved in her construction of representations. The child's "personal relationship" to her representation is incidental.

Representation is somewhat different in the constructivist approach. The child is assumed to be active. However, there is an attempt to model, not only the child's actions, but also the child's creations of objects. In other words, the constructivist attempts to explain the origin or genesis of a child's representations. In order to do this, the constructivist studies not only the child's present activity but also her developmental history. The learner is active in a "wider sense". The goal is not just to describe the child's activity in terms of a formal model but also to describe and explain what the child is active with during the process. One might say that the constructivist attempts to make the child's action "meaningful" by attempting to explain why a child behaves in a particular way. Cobb argues

that constructivists attempt to see the activity from the child's point of view (Cobb, 1987).

To summarize: Both the IP1 theorist and the constructivist assume that the mind is an active in its creation of new knowledge. However, the IP1 theorist attempts to imitate this activity with a model ("Darstellung"), whereas the constructivist attempts to understanding how the child represents or constructs things ("Vorstellungen") as she learns. The declarative network plus production rule framework of the IP theorist must "serve two masters": the machine that will ultimately simulate it and the child's cognitive processes which the machine simulates. The constructivist, on the other hand, has only the child's mental activity to account for. The constructivist might argue that she attempts to explain the natural genesis of representations, rather than attempting to cast these representations in a formal "straightjacket" that can be modeled on a machine. The IP1 theorist might counter that the constructivist is attempting to do the impossible: To explain not only the function but also the structure of inner processes. This will be examined in more detail in the next section.

- <u>functional versus structural description of the mind</u>

As discussed above, the IP1 theorist views the mind as a processor of information. "What the mind is" is explained by detailing "what it does" (its function). More specifically, proponents of this functional theory of mind construct mental

activity from three components: inputs from the environment, outputs to the environment, and "internal" (or logical) states. As Churchland describes it, functionalism is the view that:

> ... the essential or defining feature of any type of mental state is the set of causal relations it bears to (1) environmental effects on the body, (2) other types of mental states, and (3) bodily behavior... (Churchland, 198-, p. 36.)

The word "causal" here is tricky. Causal is more in the logical sense rather than the material sense. In Aristotelian terms, it is formal rather than efficient causation. Mental states are essentially logical states defined in terms of inputs and outputs. The content of these mental states is exhausted by the functional role played by these states. The model "hangs together" to the extent that the internal states play a functional or systematic logical role in the explanation of behavior. In other words, these states must be somehow "responsible" for the output, given the input.

A functional account of mental processes does not address the question: "What is really happening inside the mind?" Functionalism does not pretend to "get inside the learner's head", beyond specifying the logical relationships among its internal states. As is repeatedly pointed out in discussions of functionalism, there might be multiple material ways in which the same functional state is realized (Anderson, 1976; Churchland, 1986; Phillips, 1983). An alien being or a computer might react

to the same sets of inputs and produce the same outputs as the human. Yet, these beings might be made of very different "stuff" from the human. The nature of internal representations ("what is inside the black box") cannot be determined from the behavioral data (Kieras, 1980; Phillips, 1983). As Wittgenstein observed: "An 'inner process' stands in need of outward criteria." (Wittgenstein, 1968, part I, section 580).

The functional theory cannot uniquely determine the nature of inner representation. Nonetheless, the question: "What is inside the 'black box'?", is not easily dismissed. This might be due, in part, to the longstanding belief that we have privileged access to what is happening inside our minds. From the learner's point of view, the black box contains representations that she constructs and which become the objects of her knowledge. What is the nature of these representations?

IP1 theorists such as Anderson take Wittgenstein's caution to heart. Anderson grants that there is no way to determine, uniquely, the nature of these internal representations. He even provides a mathematical proof show that internal structures and processes cannot be uniquely determined (Anderson, 1976, pp. 10-14). However, he argues that, on grounds of theoretical parsimony, effectiveness, and broad generality that the propositional representations of knowledge are superior to current alternatives (ibid., pp. 15 ff). Similarly, Norman argues that:

> the study of propositional representations: the studies of semantic networks and schemas... is the most complete, sophisticated, and, in my opinion, successful [study]... (Norman, 1982, p,. 64).

Propositional representations are "functional" in the sense that they "are ideally suited form manipulation as computer data structures..." (Norman, 1982, p. 64). In other words, propositional representations serve the purposes or "functions" of the IPi theorist who constructs the model.

When IPi theorists speak of "productions" and "propositional nets", they are speaking about representation as "pictures of knowledge" or as "Darstellungen". However, in the interest of theoretical functionality (parsimony, effectiveness, broad generality), the implicit assumption seems to be made that this picture of knowledge is also the object of knowledge, from the learner's point of view. In other words: Though the "black box" is inscrutable, IPi theorists seem to assume that the black box contains semantic nets and production rules. The hypothesis is that the mental representation is similar to the explanatory or "model" representation. In other words, knowledge inside the child's head is represented in propositional form.

This position is probably rejected by most constructivists and is explicitly rejected by Piaget. According to Piaget, propositional representations are a larger class of semiotic representations and are proceeded by representations in actions (Piaget, 1966). "Objects of knowledge", for Piaget, are not

propositional structures but are internalized actions. Piaget explains how the child constructs her internal or mental representations by a process of reflective abstraction.

Piaget's representations are "functional" in a different sense from which IP representations are "functional". In Piaget's functionalism, "Why something is" is not explained in terms of its inputs and outputs, at least directly. Rather, it is explained in terms of its larger purpose for helping the organism to survive. A representation is "functional", in Piaget's sense, if it helps the learner come to grips with the environment. The genetic model can be understood as an attempt to explain, not just the functioning of the mind and the internal states that are responsible for the functioning, but also the origin or genesis of these structures. "Functional" is taken in a wider sense so that it means not only "preserving the data" (flow of information) but also the entire developmental history.

According to Piaget, a structure has a particular function which leads to a new structure (Piaget, 1966). This functionalist-structuralist theory relies on an every increasing ability of the organism to obtain a type of equilibrium. The change to a more complex structure must somehow enhances this ability. Here, "functional" is being used not in the input-output sense but more in the teleological sense (Reese & Overton, 1970). "What something is" is explained by what it is "supposed to do" and by then laying out the structures that help it reach these goals. Thus, Piaget insists that a study of structures

must also include a study of their genesis (Piaget, 1966). Stages and growth are taken as hard core assumptions, just as reduction is a hard core assumption in many information processing models (Reese & Overton, 197).

Piaget attempts to describe representation as an "object of knowledge". In other words, he attempts to describe the structure of the functional entities. These structures arise from the constructive process of reflective abstraction on sensori-motor activity. Through the help of negative feedback, the structures are modified so that they "fit" the purpose for which they were constructed. Piaget also hypothesizes that internal representations conform to a certain pattern: that of a mathematical group. In other words, he argues that the mental representations ("Vorstellungen") also be pictured or represented as a mathematical group. The logico-mathematical group, like the semantic net and production rules of the IP1 theorists, is a "Darstellung". Just why these patterns are favored is a problem with Piaget's theory (Phillips, 1987).

- <u>the relationship between logic and psychology</u>

Many of the differences between the IP1 and the constructivist views of representation can be traced to different views on the relationship between declarative and procedural knowledge. The present section will argue that the declarative versus procedural issue is related to a more fundamental issue pertaining to the relationship between logic and psychology.

The distinction between propositional and procedural knowledge, or between "knowledge that" and "knowledge how", can be supported, in part, by an appeal to the nature of knowledge. Karl Popper's work is useful here. Popper argues that there are three, metaphysical realms: World 1, of physical objects, World 2, of mental states, and World 3, of "objective contents of thought" (Popper, 1972). World 3 is a product of human activity, and thus its origin might involve both declarative and procedural knowledge. However, once a piece of "human knowledge" is produced, it can be expressed in an objective form.

Popper explains this notion of "objective knowledge" using a biological analogy. Animals often produce nonliving structures, such as "spiders' webs, or nests built by wasps or ants, the burrows of badgers, dams constructed by beavers, or paths made by animals in forests." (Popper, 1972, p. 112). Similarly, humans produce objective knowledge, which includes the contents of books, journals, and libraries but most important (for Popper) "theoretical systems", "problems", "problem situations", and "critical arguments" (Popper, 1972, p. 107). World 3 knowledge may have been originally born in action. However, this knowledge can be represented in books, journals, libraries, and theoretical systems. In other words, World 3 knowledge can be expressed in propositional form.

Knowledge represented in propositional form might be regarded as "finished knowledge" rather than knowledge in action. (Dewey, 1916; Romberg and Carpenter, 1986). Dewey argues that

too often this "record of knowledge, independent of its place as an outcome of inquiry and as a resource in further inquiry, is taken to be knowledge..." (Dewey, 1916, pp. 186-87). Similarly, Romberg and Carpenter trace many of the problems of mathematics instruction to the confusion between "record of knowledge" and knowledge in action (Romberg and Carpenter, 1986).

Constructivists favor the active representation of knowledge over the propositional representation. In Piaget's theory, for example, knowledge seems to be a species of the genus "activity". Piagetian knowledge would seem to be better regarded as a verb rather than as a noun.

The view that knowledge is closely tied to action, however, can be countered by an argument that distinguishes between the origin and the justification of a knowledge claim. One might readily grant, with Piaget, Dewey, and many others, that knowledge originates in the activity of human beings. But this does not imply that knowledge is essentially the same as activity. One of the fundamental features of knowledge, it might be argued, is that knowledge claims are capable of being justified or at least supported. The support for a knowledge claim is not the same as the origin of a knowledge claim. A young child might have come to believe that 7 + 5 = 12 for any number of reasons. However, what makes this believe "knowledge" is not how the child came to believe that 7 + 5 = 12, but rather the support that this statement receives from certain properties of our number system.

What is at issue here is sometimes referred to as "the genetic fallacy", or "psychologism". The question is: What is the relationship between the psychological origin of a mathematical belief and the "truth" of that belief? Constructivists would generally argue that the truth and the origin are very closely connected. As von Glasersfeld puts it, the notion of truth in the constructivist framework is one of "fitting" rather than "matching" (von Glasersfeld, 1984). The origin of a belief is intimately connected with the tendency of the belief to help an organism or a human learner cope with her environment. In other words, the origin of a belief is connected with its truth.

The counter argument would be that validity or truth is not necessarily connected with origin. It is not possible to determine the truth of a given statement by examining the psychological and genetic facts that lead to its origin. Questions of psychology and questions of logic are different. The former pertains to origin, and the latter pertains to a systematic fit between a given statement and other sets of statements. From a logical point of view, a statement is justified by a series of other statements in the form of a propositional argument. In other words, though knowledge might "originate" in action, what makes a given statement a bit of "knowledge" does not depend on where it came from but rather on how it is supported by some sort of linguistic argument. Knowledge can be expressed in the form of some sort of

proposition.

The issue that separates the IP1 theorists and the constructivists pertains to the representation of knowledge inside the learner's mind. The IP1 representation takes more stock in the propositional nature of knowledge, whereas the constructivist emphasizes the activite origin of all human beliefs.

From an logical point of view, there may be strong reasons for viewing knowledge in declarative or propositional form. Justification requires an argument, which must be statable in some form of linguistic argument. This conception of knowledge serves certain purposes, which match many of those who wish to construct formal or logical models of human information processing. From a psychological point of view, however, there may be equally strong reasons for viewing knowledge as originating in activity. The evidence seems to point to the fact that the child acquire her knowledge of the world by operating on it. This representation of knowledge serves certain purposes, which match those who wish to explain the origin of knowledge and also to help children construct their own mathematical concepts.

Conclusion: Function and Representation

Representations must serve a certain purpose. There is no need to send a representative if the real thing will do the job. If it were managerially feasible for every member of the United States to vote in Congress on every issue, the House of

Representatives would not be necessary. A representation often serves the purpose of functioning in the absence of the real thing. In the absence of being able to send human beings off to distant planets, NASA encodes a "representation" of the human species, in iconic form, on probes to be sent off into space.

Symbols are used to represent words and ideas in human language. These symbols "function" in the sense that they make possible communication across time and space. The ability to encode images and ideas in language makes it unnecessary to be standing face to face with someone to communicate. According to Piaget, however, even if one were not interested in communicating with another person, representations still play an essential function in thought. Namely, for levels of thought above mere "reacting", representations enable the human being to, as it were, carry on a conversation with him or herself. More specifically, the human learner represents to herself past actions which are then mentally manipulated as objects to create systems of mental operations. The ability to replay or re-present these past actions enables one to reflect on things that once were or which might be (hypothetical thought).

Thus, representations "function" in various senses. In one sense, they function in the sense of preserving records of the past (eg., printed words in ancient books). In another sense, they function like communication "stunt men" that can go where the real ideas cannot go. In another sense, they function in the sense of permitting the recall of things past or possible for

analysis.

This paper has examined different senses in which representations "function" in constructivist and information processing frameworks in mathematics education. For IP theorists, representations "function" in the sense that they can explain what outputs a child will produce to a problem, given a particular input. For constructivists, representations "function" in the sense that they help a knower to organize her sensory input and her previous knowledge into a coherent scheme.

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